Journal of Forestry Research and Management. Vol. 15(2).33-43; 2018, ISSN 0189-8418
www.jfrm.org.ng

Comparison of Gamma, Lognormal and Weibull Functions for Characterising Tree Diameters in Natural Forest

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ABSTRACT

The performance of distribution function depends to a large extent on the method of estimating its parameters. A poor estimation method will invariably yield a poor result; in consequence, mislead the forest owner. Therefore, in this study, three estimation methods were used to fit the gamma, lognormal and Weibull functions to the diameter distributions of Ikrigon forest reserve. Data were collected from 10 temporary sample plots of 0.25 ha size. The plots were established using systematic sampling technique. Trees with diameter at breast height (dbh) = 10 cm were measured. Three estimation methods including maximum likelihood estimation (MLE), minimum distance estimation (MDE) weighted by Kolmogorov-Smirnov and percentile methods were used. Assessment was based on both graphical and fit indices including Cramer-von Mises (W^2) statistic, Anderson-Darling (A-D) statistic, Kolmogorov-Smirnov (K-S) statistic, Akaike information criterion (AIC) and Bayesian information criterion (BIC). The results showed that MDE and percentile methods improved the fits of the gamma, lognormal and Weibull distributions than MLE. The lognormal distribution had the smallest W^2, A-D, K-S, AIC and BIC values of 0.098, 0.812, 0.069, 623.074 and 627.726 for MDE and 0.115, 0.926, 0.096, 624.907 and 629.559, for percentiles, respectively. This was followed by gamma and Weibull distributions. The MDE and percentiles methods are recommended for fitting the functions to tree diameter data. The methods can be used for other functions to describe the structure of forest stand, especially plantations.

Keywords: Diameter distributions, distribution functions, distance estimation, Ikrigon forest reserve

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INTRODUCTION

Tree diameter distribution is the graphical or tabular representation of the total number of tree by diameter classes. It is the basis for assessing forest stand structure, volume production and stability (Gorgoso-Varela and Rojo-Alboreca, 2014). Diameter distribution modelling is an effective tool for growth and yield studies, and as such, can be used for forest management and decision making. The study on diameter distribution was pioneered by De Liocourt (1898) when the author reported the inverse J-shaped of natural forest.

In recent times, several statistical functions have been used to describe the structure of both natural and plantation forests. Notably among these functions are the beta (Maltamo *et al*., 1995, Gorgoso *et al*., 2012; Ige *et al*., 2013; Ogana *et al*., 2015), Johnson's S_B (Knoebel and Burkhart, 1991; Zhou and McTague, 1996; Zhang *et al*., 2003; Gorgoso-Varela and Rojo-Alboreca, 2014; Ogana *et al*. 2017), gamma (Nelson, 1964; Zheng and Zhou, 2010; Ogana *et al*., 2015), Logit-Logistic (Wang and Rennolls, 2005; Ogana *et al*., 2018), lognormal (Mohammad *et al*., 2009; Fallahchai *et al*., 2012), and Weibull distributions (Bailey and Dell, 1973; Nanang, 1998; Palahi *et al*., 2007; Gorgoso *et al*., 2012; Ajayi *et al*., 2013; Ogana and Gorgoso-Varela, 2015). Of these distributions, the Weibull distribution is the most widely used function in forestry to describe the structure of forest stand, this is because of its relatively simplicity. The choice of a distribution depends on the following criteria including flexibility, simplicity of estimation proportion in diameter classes and simplicity of parameter estimation method (Burkart and Tomé, 2012).

The accuracy of a distribution function to characterised the structure of forest stand depends on the method of estimation. Different estimation methods have been proposed and used to fit distribution functions to tree diameter data including maximum likelihood, moment, percentiles, least square estimation methods, etc. While most of these methods have been successfully used for the Johnson's S_B and Weibull distributions to characterise tree diameter (Zhou and McTague, 1996; Zhang *et al*., 2003; Gorgoso-Varela and Rojo-Alboreca, 2014), inadequate information exists in forestry literature where maximum likelihood and percentiles were used for fitting the gamma distribution. Similarly, there are dearth of studies on the lognormal distribution fitted with percentile in forestry. The gamma and lognormal are flexible enough to describe positive skew diameter distribution data, that is, a long-tail to the right. This is a common future of natural forest (inverse J-shaped).

Another method that seems not to have been applied to forestry to estimate the parameters of gamma and lognormal distributions is the minimum distance estimation (MDE) method. The MDE is also called the maximum goodness-of-fit estimation (Dutang *et al*., 2008). This method is like the least square estimation method except that a weight is used to minimized the distance between the empirical distribution and the model distribution. This is an estimation method “that gives more weight to data at one tail of the distribution” (Delignette-Muller and Dutang, 2015). Some of the goodness-of-fit criteria that have been used to minimise distance estimation include Anderson-Darling, Chi-square, Cramer-von Mises and Kolmogorov-Smirnov criteria (Parr and Schucancy, 1980). There is no rationale for the preference of one criterion over the other.

Therefore, the purpose of this study was to evaluate the suitability of different estimation methods for the gamma and lognormal for describing the stand structure of natural forest. And to compare the result with the commonly used Weibull distribution.

METHODOLOGY

Data

The data for this study were obtained from Ikrigon forest reserve. The reserve is in the northern part of Cross River State, Nigeria. The reserve was established in 1926. It lies between latitude 6°17.597' - 6°17.862'N and longitude 8°35.597' - 8°35.276'E and occupies an area of 542.7ha (Adeniyi, 2017). A systematic sampling technique was used in demarcating 10 temporary sample plots (TSPs) of 50m x 50m size. The plots were established at alternate position along the transect line at 100m interval. Only trees with diameter at breast height (dbh) = 10 cm were measured. The descriptive statistics of the stand variables are presented in Table 1.

Table1. Descriptive statistics of the stand variables

Stand Variable	Statistics			
	Mean	Max	Min	Standard deviation
dbh (cm)	29.9	125.0	10.0	15.86
Quadratic mean (cm)	33.81	39.59	29.38	3.46
Dominant height (m)	31.93	35.93	27.56	2.53
Density (N/ha)	310.4	440.0	184.0	72.82
Basal area (m^2/ha)	28.01	40.91	16.94	8.29
Volume (m^3/ha)	814.13	1235.85	447.89	264.42

Model specification

Three distributions were considered in this study including gamma, lognormal and Weibull distributions. Only distributions with 2-paramters were selected; as such provided good platform for comparison. Their probability density functions (pdf) are expressed as:

Gamma pdf $$f(x) = \frac{x^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)}\exp\left(-\frac{x}{\beta}\right) \quad (1)$$

Lognormal pdf $$f(x) = \frac{\exp\left(-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2\right)}{x\sigma\sqrt{2\pi}} \quad (2)$$

2-parameter Weibull pdf (Weibull, 1951): $$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \quad (3)$$

Where α and β are the shape and scale parameters, respectively of the gamma and Weibull distribution; μ and σ are the location and scale parameters of the lognormal function. Γ is the gamma function, x = diameter at breast height (dbh). The gamma and lognormal distributions do not have a closed-form cumulative distribution function (cdf); as such they can only be solved by numerical integration. However, the cdf of the Weibull exist and in closed-form which facilitate the estimation of proportion of trees in dbh classes.

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \quad (4)$$

All parameters are previously defined.

Estimation method and model assessment

Three estimation methods were used to fit the distributions to the diameter data. These include: maximum likelihood estimation (MLE), percentiles and the minimum distance estimation (MDE) methods. For the MDE, Kolmogorov-Smirnov was used as the weight to minimize the distance between the empirical distribution and the distribution models wherein the parameters of the distributions were estimated. In this method, the supremum of the absolute difference between empirical distribution and the model distribution was used as the weight.

The distributions were assessed based on Cramer-von Mises (W^2), Anderson-Darling (A-D) and Kolmogorov-Smirnov (K-S) statistics. Also, Akaike information criterion (AIC) and Bayesian information criterion (BIC) were used to assess the distributions. The smaller the values are, the better the model. The distributions were fitted using ‘fitdistrplus package’ (Delignette-Muller and Dutang, 2015) in R software (R Core Team, 2017).

RESULTS AND DISCUSSION

The performance of gamma, lognormal and Weibull distributions fitted with maximum likelihood, minimum distance estimation (MDE) and percentile for characterising the diameter distribution in Ikrigon forest reserve have been evaluated. The graphical representation of the relative frequency of tree per diameter classes are shown in Fig. 1 to 3. A representative sample plot (plot 10) was used to show how well these methods improved the performance of the distributions. From the graph, it can be observed that the method of MDE and percentile improved the fits of gamma, lognormal and Weibull distributions compared to method of maximum likelihood estimation (MLE). There were little variations between the observed (white bar) and estimated relative frequency of trees for MDE and percentile. The Weibull distribution fitted with MLE greatly underestimated the relative frequency in the smallest diameter class (dbh < 20 cm). Nevertheless, the three methods approximate the usual structure of natural forest (i.e., the inverse J-shaped); larger proportion of trees in the lower diameter classes with a decreasing frequency as the diameter increases.

Furthermore, five indices were used to assess the fits of these distributions vis-a-viz the different estimation methods. The mean values of the Cramer-von Mises (W^2) statistic, Anderson-Darling (A-D) statistic, Kolmogorov-Smirnov (K-S) statistic, Akaike information criterion (AIC) and Bayesian information criterion (BIC) are presented in Table 2. The result showed that lognormal distribution fitted with MDE, percentile and MLE had the smallest values for the fit indices and as such ranked the best. This was followed by the gamma distribution. The Weibull distribution ranked the least. Its values were larger than lognormal and gamma distributions.

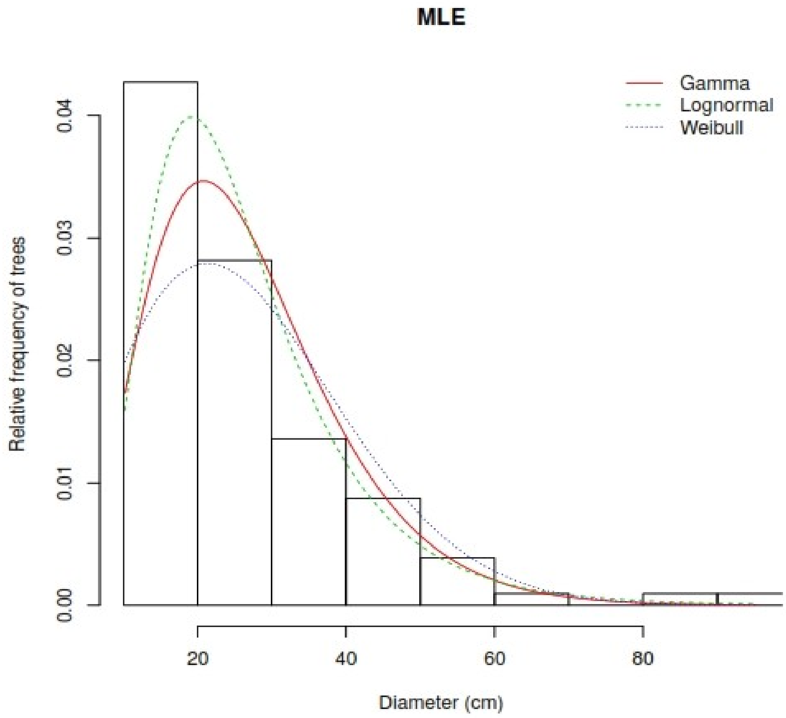

Fig. 1: Observed and fitted gamma, lognormal and Weibull distributions by method of maximum likelihood (MLE)

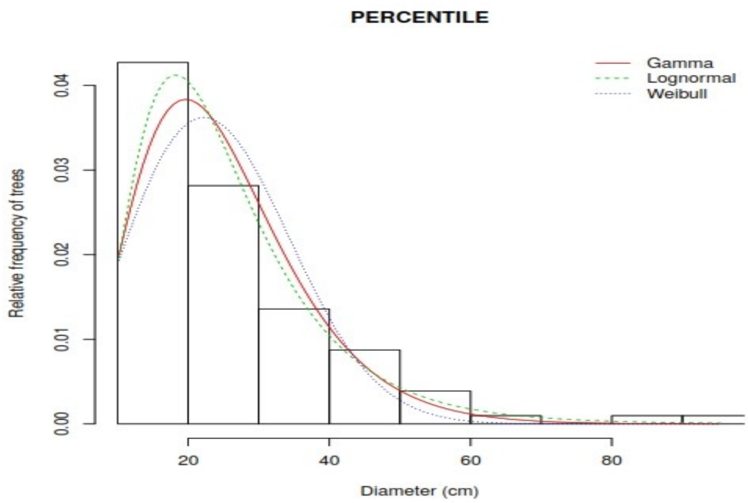

Fig. 2: Observed and fitted gamma, lognormal and Weibull distributions by method of percentile

Journal of Forestry Research and Management. Vol. 15(2).33-43; 2018, ISSN 0189-8418
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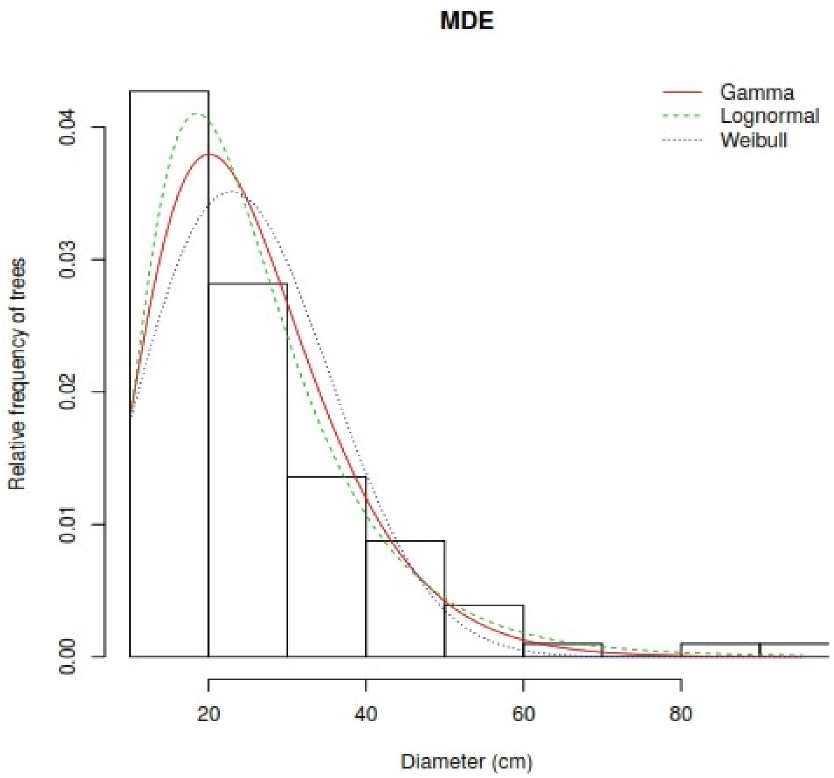

Fig. 3: Observed and fitted gamma, lognormal and Weibull distributions by method of minimum distance estimation (MDE)

Table 2. Mean fit indices for the gamma, lognormal and Weibull distributions

Estimation methods	Fit indices	Distribution functions		
		Gamma	**Lognormal**	**Weibull**
MLE	K-S	0.098	0.096	0.106
	W^2	0.158	0.135	0.192
	AD	1.008	0.856	1.301
	AIC	625.199	621.864	631.737
	BIC	629.851	626.516	636.390
Percentile	K-S	0.100	0.096	0.109

	W^2	0.123	0.115	0.147
	AD	1.055	0.926	1.491
	AIC	627.862	624.907	636.965
	BIC	632.514	629.559	641.617
MDE	K-S	0.073	0.069	0.082
	W^2	0.117	0.098	0.162
	AD	1.047	0.812	1.675
	AIC	627.143	623.074	640.302
	BIC	631.795	627.726	644.954

MDE = minimizing distance estimation; K-S = Kolmogorov-Smirnov; W^2 = Cramer-von Mises; AD = Anderson-Darling; AIC = Akaike information criterion; BIC = Bayesian information criterion

This study is in tandem with Nanang (1998) who reported a better performance for the lognormal distribution than the Weibull for age group estimation. Similar result was observed in Sheykholeslami *et al*. (2011) who also found the lognormal distribution to be suitable than gamma and Weibull distributions in modelling the diameter distribution in natural forest. However, Mohammed *et al*. (2009) found the gamma distribution to be more appropriate than the exponential and lognormal distributions. These authors used the method of maximum likelihood to fit the lognormal and Weibull distributions. In this study, minimum distance estimation and percentile were equally good as the maximum likelihood for the gamma, lognormal and Weibull distributions. Most studies reported in forestry literature on gamma distribution used the method of moments for the gamma distribution (Ogana *et al*., 2015). Better performance was observed for the gamma distribution fitted with maximum likelihood, MDE and percentile than the much-used method of moment. Furthermore, two percentile points that is, 25% and 75% were used for the percentile estimation method in this study. Better results were obtained with these points than the 17% and 97% percentiles recommended by Dubey (1967) for the Weibull distributions.

The characterisation of tree diameter distribution provides the basis for assessing not only the forest stand structure but also the regeneration potential of the forest. For example, the forest structure represented in the figures above indicate that there are potential trees in the lower diameter class that can grow into the larger diameter classes if the ecosystem is not disturbed. More so, the graph provides information on the proportion of trees that can be readily harvested based on a specified merchantable limit. This information is needed for routine forest inventory and sustainable forest management. In this study, three commonly used 2-parameter distributions were used to derive such information. The lognormal and gamma distributions were more consistent than the Weibull distributions.

CONCLUSION

Modelling diameter distribution requires the identification of suitable function and appropriate estimation method that will yield the best estimate for the parameters of the candidate function. In this study, minimum distance estimation (MDE) weighted by Kolmogorov-Smirnov, percentile and maximum likelihood estimation methods were found to be suitable for the gamma and lognormal distributions. These methods can be used for the functions to describe the structure of the forest stand in Ikrigon forest reserve, and can also be extended to other forest stand especially plantations.

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